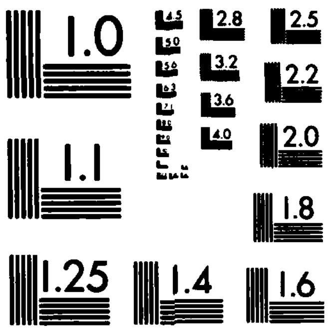
AD-A136 768 NONLINEAR DYNAMIC POLARIZATION FORCE ON A RELATIVISTIC TEST PARTICLE IN A NONEQUILIBRIUM BEAM-PLASMA SYSTEM (U) HARRY DIAMOND LABS ADELPHI MD H E BRANDT SEP 83 1/1

UNCLASSIFIED HDL-TR-1994 F/G 20/8 NL

END



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(12)

HDL-TR-1994

September 1983

Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System

by Howard E. Brandt

U.S. Army Electronics Research and Development Command
Harry Diamond Laboratories
Adelphi, MD 20783

... release; distribution unlimited.

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Citation of manufacturers' or trade names does not constitute an official indorsement or approval of the use thereof.

Destroy this report when it is no longer needed. Do not return it to the originator.

A preliminary version of this report was issued as HDL-PRL-82-6 in May 1982.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER HDL-TR-1994	2. GOVT ACCESSION NO. AD-A136768	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Howard E. Brandt		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Harry Diamond Laboratories 2800 Powder Mill Road Adelphi, MD 20783		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Ele: 61101A
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Materiel Development and Readiness Command Alexandria, Va 22333		12. REPORT DATE September 1983
		13. NUMBER OF PAGES 30
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

HDL Project: A10225
DRCMS Code: 1FOR0007011FA9
DA Project: 1L161101A91A

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Radiation	Relativistic electron beams
Microwaves	Radiative instability
Plasma turbulence	Bremsstrahlung
Relativistic plasma	Beam-plasma systems
Plasma physics	Nonlinear plasmas

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The effective charge of a test particle in a nonequilibrium beam-plasma system is the sum of the actual charge and the dynamic polarization charge surrounding it. The contribution of its polarization charge to the total force on a relativistic test particle, namely the dynamic polarization force, is calculated to fourth order in the total electromagnetic field for a slowly varying, nearly spatially independent background distribution with no external fields. This relation is needed in calculations of collective radiation processes and the conditions for the occurrence of radiative instability in relativistic beam-plasma systems.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

CONTENTS

Page

1. INTRODUCTION 5

2. THE DYNAMIC POLARIZATION CHARGE DENSITY 6

3. THE DYNAMIC POLARIZATION CURRENT DENSITY 12

4. THE NONLINEAR DYNAMIC POLARIZATION FORCE 17

5. CONCLUSION 21

LITERATURE CITED 22

DISTRIBUTION 23

1. INTRODUCTION

In a nonequilibrium beam-plasma system, the time-average nonlinear force $\langle\vec{F}_\alpha\rangle$ on a relativistic test particle due to the total electromagnetic field consists of a part $\langle\vec{F}_{\alpha a}\rangle$, acting on the actual charge of species α, and a part $\langle\vec{F}_{dp}\rangle$, acting on the dynamic polarization charge surrounding the particle. Thus,

$$\langle\vec{F}_\alpha\rangle = \langle\vec{F}_{\alpha a}\rangle + \langle\vec{F}_{dp}\rangle \quad . \tag{1}$$

The polarization charge arises from the redistribution of other particles due to the induced fields produced in the neighborhood of the particle as it moves through the beam-plasma. For a relativistic test particle the time-average force on its actual charge (bare charge) under conditions of the Born approximation for plasma is given by[1,*]

$$\langle\vec{F}_{\alpha a}\rangle = \vec{F}_\alpha^{(1)} + \vec{F}_\alpha^{(2)} \quad , \tag{2}$$

where

$$\vec{F}_\alpha^{(1)} = \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int dk\, \vec{k}\, \frac{\vec{v}_\alpha\cdot\vec{E}_k}{\omega + i\delta}\, \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \tag{3}$$

and

$$\vec{F}_\alpha^{(2)} = \lim_{t\to\infty} \frac{\pi i}{t} e_\alpha \int dk\, dk_1\, \frac{\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\alpha\right)}{(\omega + i\delta)(\omega_1 + i\delta)}$$

$$\times E_{ki}E_{k_1 j}\left[\vec{k}\Lambda_{ij}^{(\alpha)}(k_1,k) + \vec{k}_1\Lambda_{ij}^{(\alpha)*}(k_1,k)\right] \tag{4}$$

and

[1] A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].

*H. E. Brandt, Nonlinear Force on an Unpolarized Relativistic Test Particle to Second-Order in the Total Field in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-82-7 (May 1982), to be published as HDL-TR-1995.

$$\Lambda_{ij}^{(\alpha)}(k_1,k) = \frac{e_\alpha}{\gamma_\alpha m_\alpha}\left[\delta_{ij} + \frac{v_{\alpha i}k_j - v_{\alpha j}k_{1i}}{\omega - \vec{k}\cdot\vec{v}_\alpha - i\delta} - \frac{v_{\alpha i}v_{\alpha j}}{(\omega - \vec{k}\cdot\vec{v}_\alpha - i\delta)^2}\left(\vec{k}\cdot\vec{k}_1 - \frac{\omega\omega_1}{c^2}\right)\right] . \tag{5}$$

Here t is the time, e_α is the actual charge of the particle of species α, m_α is its mass, $k = (\vec{k},\omega)$ is a wave four-vector, δ is a small imaginary part of the frequency, $\delta(x)$ is the Dirac delta function, $E_{\vec{k}i}$ is the Fourier transform of the i^{th} component of the total electric field, $\vec{v}_\alpha$ is the particle velocity, $\gamma_\alpha = (1 - v_\alpha^2/c^2)^{-1/2}$, and c is the speed of light. The condition that equation (2) hold, namely, the Born approximation, is that

$$\frac{e_\alpha|\vec{E}_\alpha|}{\omega_{pe}|\vec{p}_\alpha|} << 1 \quad , \tag{6}$$

where $\vec{p}_\alpha$ is the particle momentum and ω_{pe} is the electron plasma frequency.

In the present work, the time-average of the dynamic polarization force $\langle\vec{F}_{dp}\rangle$ is derived to fourth order in the total field for a slowly varying nearly spatially independent background with no external fields. It is shown to be dependent on the linear, second- and third-order nonlinear conductivity tensors. The result of this calculation agrees with that of Akopyan and Tsytovich.[1] It is important in calculations of collective radiation processes and the conditions for the occurrence of radiative instability in nonequilibrium beam-plasma systems.

2. THE DYNAMIC POLARIZATION CHARGE DENSITY

The dynamic polarization charge which surrounds the test particle results from the background distribution function being disturbed in the neighborhood of the particle. Thus, the associated current density is given by

$$\vec{j}_{dp} = \sum_s e_s \int \frac{d^3\vec{p}_s}{(2\pi)^3}\,\vec{v}_s\left(f^{(s)} - f^{R(s)}\right) \quad , \tag{7}$$

where the sum is over all species and $f^{(s)}$ and $f^{R(s)}$ are the perturbed and regular background distribution functions, respectively, for species s. It is assumed that there are no external fields and the perturbation in the distribution is due to the field associated with the interaction between the test

[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

particle and the other particles in the system. The dynamic polarization charge density is related to the current in equation (7) by the equation of continuity, namely,

$$\frac{\partial \rho_{dp}}{\partial t} + \vec{\nabla} \cdot \vec{j}_{dp} = 0 \quad . \tag{8}$$

Equation (8) assumes local conservation of the dynamic polarization current. The Fourier decompositions of the polarization charge and current densities are given by

$$\rho_{dp} = \int dk \; \rho_{dpk} e^{i(\vec{k}\cdot\vec{r}-\omega t)} \tag{9}$$

and

$$\vec{j}_{dp} = \int dk \; \vec{j}_{dpk} e^{i(\vec{k}\cdot\vec{r}-\omega t)} \quad , \tag{10}$$

where

$$dk \equiv d^3\vec{k} \; d\omega \quad . \tag{11}$$

Using equations (9) and (10) in equation (8), then

$$\rho_{dpk} = \frac{\vec{k}\cdot\vec{j}_{dpk}}{\omega + i\delta} \quad . \tag{12}$$

Taking the Fourier transform of equation (7) one has also

$$\vec{j}_{dpk} = \sum_s e_s \int \frac{d^3\vec{p}_s}{(2\pi)^3} \left(f_k^{(s)} - f_k^{R(s)}\right)\vec{v}_s \quad . \tag{13}$$

Substituting equation (13) in equation (12) one obtains the following expression for the Fourier transform of the dynamic polarization charge density:

$$\rho_{dpk} = \sum_s e_s \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta} \left(f_k^{(s)} - f_k^{R(s)}\right) \quad . \tag{14}$$

The particle distribution functions $f^{(s)}$ are determined by the relativistic Vlasov equation, namely,

$$\partial_t f^{(s)} + \vec{v}_s\cdot\vec{\nabla}_{r_s} f^{(s)} + \vec{F}_s\cdot\vec{\nabla}_{p_s} f^{(s)} = 0 \quad , \tag{15}$$

where the relativistic relation between velocity and momentum is given by

$$\vec{v}_s = \left[1 + \left(\frac{p_s}{m_s c}\right)^2\right]^{-1/2} \frac{\vec{p}_s}{m_s} \tag{16}$$

and the force $\vec{F}_s$ is given by

$$\vec{F}_s = e_s\left(\vec{E} + \vec{v}_s \times \vec{B}\right) \quad . \tag{17}$$

Here $\vec{E}$ and $\vec{B}$ are the total electric and magnetic fields. The distribution function and the fields can be expressed in terms of their Fourier transforms. Thus, for example,

$$f^{(s)} = \int dk \; f_k^{(s)} e^{i\left(\vec{k}\cdot\vec{r}_s - \omega t\right)} \quad . \tag{18}$$

In terms of the Fourier transforms, equation (15) becomes

$$f_k^{(s)} = \frac{1}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \int dk_1 \; dk_2 \delta(k - k_1 - k_2) \vec{F}_{sk_1} \cdot \vec{\nabla}_{p_s} f_{k2}^{(s)} \quad . \tag{19}$$

Expressing the distribution functions as power series in the total field, one has

$$f_k^{(s)} = f_k^{R(s)} + \sum_{n=1}^{\infty} f_k^{(s)(n)} \quad , \tag{20}$$

where $f_k^{R(s)}$ describes the background for the nonequilibrium beam-plasma. The $f_k^{s(n)}$ describe the perturbation in the background due to the total electromagnetic field. Assuming a slowly varying, nearly spatially independent background which in zeroth approximation is space and time independent and denoted by $f_{p_s}^{R(0)}$, then

$$f_k^{R(s)} = (2\pi)^{-4} \int d^3\vec{r} dt \; f_{p_s}^{R(0)} e^{-i\left(\vec{k}\cdot\vec{r} - \omega t\right)} \quad . \tag{21}$$

Performing the integration in equation (21), then

$$f_k^{R(s)} = f_{p_s}^{R(0)} \delta(k) \quad , \tag{22}$$

where $\delta(k) \equiv \delta^3(\vec{k})\delta(\omega)$ is the four-dimensional Dirac delta function. For stationary turbulence, equation (22) is exact. It follows from equations (19) and (20) by iteration that

$$f_k^{(s)(1)} = \frac{1}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \int dk_1 dk_2\ \delta(k - k_1 - k_2)\vec{F}_{sk_1}\cdot\vec{\nabla}_{p_s} f_{k_2}^{R(s)} , \tag{23}$$

$$\begin{aligned} f_k^{(s)(2)} = \frac{1}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \int & dk_1 dk_2 dk_3 dk_4\ \delta(k - k_1 - k_2)\delta(k_2 - k_3 - k_4) \\ & \times \vec{F}_{sk_1}\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk_3}\cdot\vec{\nabla}_{p_s} f_{k_4}^{R(s)} , \end{aligned} \tag{24}$$

and

$$\begin{aligned} f_k^{(s)(3)} = \frac{1}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \int & dk_1 dk_2 dk_3 dk_4 dk_5 dk_6\ \delta(k - k_1 - k_2) \\ & \times \delta(k_2 - k_3 - k_4)\delta(k_4 - k_5 - k_6)\vec{F}_{sk_1}\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta\right)} \\ & \times \vec{F}_{sk_3}\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega_4 - \vec{k}_4\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk_5}\cdot\vec{\nabla}_{p_s} f_{k_6}^{R(s)} . \end{aligned} \tag{25}$$

Substituting equation (22) in equations (23,24,25) and using the properties of the delta function to do some of the integrations, then

$$f_k^{(s)(1)} = \frac{1}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk}\cdot\vec{\nabla}_{p_s} f_{p_s}^{R(0)} , \tag{26}$$

$$\begin{aligned} f_k^{(s)(2)} = \frac{1}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \int & dk_1 dk_2\ \delta(k - k_1 - k_2)\vec{F}_{sk_1}\cdot\vec{\nabla}_{p_s} \\ & \times \frac{1}{i\left(\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk_2}\cdot\vec{\nabla}_{p_s} f_{p_s}^{R(0)} , \end{aligned} \tag{27}$$

and

$$f_k^{(s)(3)} = \frac{1}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \int dk_1 dk_2 dk_3 dk_4 \; \delta\left(k - k_1 - k_2\right)\delta\left(k_2 - k_3 - k_4\right)$$

$$\times \vec{F}_{sk_1}\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk_3}\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega_4 - \vec{k}_4\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk_4}\cdot\vec{\nabla}_{p_s} f_{p_s}^{R(0)} \quad . \tag{28}$$

The Fourier transform $\vec{F}_{sk}$ of the Lorentz force $\vec{F}_s$ is given by

$$\vec{F}_{sk} = e_s\left(\vec{E}_k + \vec{v}_s\times\vec{B}_k\right) \quad . \tag{29}$$

However, from Maxwell's equation,

$$\partial_t\vec{B} = -\vec{\nabla}\times\vec{E} \quad , \tag{30}$$

it follows that

$$\vec{B}_k = \frac{1}{\omega + i\delta}\,\vec{k}\times\vec{E}_k \quad , \tag{31}$$

and substituting equation (31) in equation (29), then

$$\vec{F}_{sk} = e_s\left[\vec{E}_k + \frac{\vec{v}_s\times\left(\vec{k}\times\vec{E}_k\right)}{\omega + i\delta}\right] \quad . \tag{32}$$

Then using the vector identity, one has

$$\vec{v}_s\times(\vec{k}\times\vec{E}_k) = \left(\vec{v}_s\cdot\vec{E}_k\right)\vec{k} - \left(\vec{v}_s\cdot\vec{k}\right)\vec{E}_k \quad , \tag{33}$$

and equation (32) becomes

$$\vec{F}_{sk} = e_s\left[\vec{E}_k\left(1 - \frac{\vec{v}_s\cdot\vec{k}}{\omega + i\delta}\right) + \vec{k}\,\frac{\vec{v}_s\cdot\vec{E}_k}{\omega + i\delta}\right] \quad . \tag{34}$$

Next substituting equation (20) in equation (14) one obtains for the Fourier transform of the dynamic polarization charge density

$$\rho_{dpk} = \sum_s e_s \sum_{n=1}^{\infty} \int \frac{d^3\vec{p}_s}{(2\pi)^3}\,\frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta}\, f_k^{(s)(n)} \quad . \tag{35}$$

Similarly, the Fourier transform of the dynamic polarization current density equation (13) becomes

$$\vec{j}_{dpk} = \sum_s e_s \sum_{n=1}^{\infty} \int \frac{d^3\vec{p}_s}{(2\pi)^3} \vec{v}_s f_k^{(s)(n)} \quad . \tag{36}$$

Equation (35) may be written as

$$\rho_{dpk} = \sum_s \rho_{dpk}^{(s)} \quad , \tag{37}$$

where

$$\rho_{dpk}^{(s)} = \sum_{n=1}^{\infty} \rho_{dpk}^{(s)(n)} \tag{38}$$

and

$$\rho_{dpk}^{(s)(n)} = \int \frac{d^3\vec{p}_s}{(2\pi)^3} e_s \frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta} f_k^{(s)(n)} \quad . \tag{39}$$

Similarly, equation (36) may be written

$$\vec{j}_{dpk} = \sum_s \vec{j}_{dpk}^{(s)} \quad , \tag{40}$$

where

$$\vec{j}_{dpk}^{(s)} = \sum_{n=1}^{\infty} \vec{j}_{dpk}^{(s)(n)} \tag{41}$$

and

$$\vec{j}_{dpk}^{(s)(n)} = \int \frac{d^3\vec{p}_s}{(2\pi)^3} e_s \vec{v}_s f_k^{(s)(n)} \quad . \tag{42}$$

Equations (37) and (40) express the Fourier transform of the dynamic polarization charge density and current density in terms of a sum over the contributions of each species. Equations (38) and (41) represent the latter as expansions in the total electric field, the n^{th} order terms of which are given by equations (39) and (42). The first three orders can be obtained by substituting equations (26) to (28) and (34) in equations (39) and (42).

3. THE DYNAMIC POLARIZATION CURRENT DENSITY

The currents $\vec{j}_{dp}^{(s)(1)}$, $\vec{j}_{dp}^{(s)(2)}$, and $\vec{j}_{dp}^{(s)(3)}$ are the linear, second-order nonlinear, and third-order nonlinear dynamic polarization current densities, respectively. The linear and nonlinear electric field dependence is given by equations (42), (26) to (28), and (34).

Proceeding to reduce the linear current one has, using equations (42) and (26),

$$\vec{j}_{dpk}^{(s)(1)} = \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{e_s\vec{v}_s}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk}\cdot\vec{\nabla}_{p_s} f_{p_s}^{R(0)} \quad . \tag{43}$$

Substituting equation (34) in equation (43) then, equation (43) becomes

$$\vec{j}_{dpk}^{(s)(1)} = \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{e_s^2\vec{v}_s}{i(\omega - \vec{k}\cdot\vec{v}_s + i\delta)} \left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta}\right) + \vec{k}\left(\frac{\vec{v}_s\cdot\vec{E}_k}{\omega + i\delta}\right)\right]\cdot\vec{\nabla}_{p_s} f_{p_s}^{R(0)} \quad . \tag{44}$$

Using the Einstein convention with implicit summation over repeated indices, equation (44) may be rewritten as

$$\vec{j}_{dpki}^{(s)(1)} = E_{k\ell} \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{e_s^2 v_{si}}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \left[\delta_{\ell m}\left(1 - \frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta}\right) + \frac{k_m v_{s\ell}}{\omega + i\delta}\right] \frac{\partial f_{p_s}^{R(0)}}{\partial p_{sm}} \quad . \tag{45}$$

Equivalently, then, the linear dynamic polarization current density is given by

$$j_{dpki}^{(s)(1)} = \sigma_{i\ell}^{(s)}(k) E_{k\ell} \quad , \tag{46}$$

where the linear conductivity tensor $\sigma_{i\ell}^{(s)}$ is given by[2]

$$\sigma_{i\ell}^{(s)}(k) = e_s^2 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}\left[\delta_{\ell m}\left(1 - \frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta}\right) + \frac{k_m v_{s\ell}}{\omega + i\delta}\right]}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \frac{\partial f_{p_s}^{R(0)}}{\partial p_{sm}} \quad . \tag{47}$$

[2] *V. N. Tsytovich, Theory of Turbulent Plasma, Consultants Bureau, Plenum Publishing Corp. (New York, 1977).*

Proceeding to reduce the second-order dynamic polarization current using equations (42) and (27), then

$$\vec{j}^{(s)(2)}_{dpk} = \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{e_s\vec{v}_s}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} dk_1 dk_2\ \delta\left(k - k_1 - k_2\right)$$

$$\times \vec{F}_{sk_1}\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk_2}\cdot\vec{\nabla}_{p_s} f^{R(0)}_{p_s} \quad . \tag{48}$$

Substituting equation (34) in equation (48), then

$$\vec{j}^{(s)(2)}_{dpk} = \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{e^3_s\vec{v}_s}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} dk_1 dk_2\ \delta\left(\vec{k} - \vec{k}_1 - \vec{k}_2\right)$$

$$\times \left[\vec{E}_{k_1}\left(1 - \frac{\vec{k}_1\cdot\vec{v}_s}{\omega_1 + i\delta}\right) + \vec{k}_1\left(\frac{\vec{v}_s\cdot\vec{E}_{k_1}}{\omega_1 + i\delta}\right)\right]\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta\right)}$$

$$\times \left[\vec{E}_{k_2}\left(1 - \frac{\vec{k}_2\cdot\vec{v}_s}{\omega_2 + i\delta}\right) + \vec{k}_2\left(\frac{\vec{v}_s\cdot\vec{E}_{k_2}}{\omega_2 + i\delta}\right)\right]\cdot\vec{\nabla}_{p_s} f^{R(0)}_{p_s} \quad . \tag{49}$$

Equivalently, then, the second-order nonlinear dynamic polarization current density, equation (49), may be rewritten as

$$j^{(s)(2)}_{dpki} = -e_s \int \frac{dk_1 dk_2\ \delta\left(k - k_1 - k_2\right)}{\left(\omega_1 + i\delta\right)\left(\omega_2 + i\delta\right)} S^{(s)}_{ij\ell}\left(k, k_1, k_2\right) E_{k_1 j} E_{k_2 \ell} \quad , \tag{50}$$

where the second-order nonlinear conductivity tensor $S^{(s)}_{ij\ell}(k,k_1,k_2)$ for species s is given by[1,3-8]

[1] A. V. Akopyan and V. N. Tsytovich, *Bremsstrahlung in a Nonequilibrium Plasma*, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].

[3] H. E. Brandt, *Symmetries of the Nonlinear Conductivity for a Relativistic Turbulent Plasma*, Harry Diamond Laboratories, HDL-TR-1927 (March 1981).

[4] H. E. Brandt, *Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma*, Phys. Fluids, 24 (1981), 1760.

[5] H. E. Brandt, *Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, in Plasma Astrophysics, Course and Workshop, Organized by the International School of Plasma Physics, 27 August to 7 September 1981, Varenna (Como), Italy, European Space Agency ESA SP-161 (1981) (also to be published by Pergamon Press), 361.

[6] H. E. Brandt, *On the Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, Harry Diamond Laboratories, HDL-TR-1970 (February 1982).

[7] H. E. Brandt, *Comment on Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma*, Phys. Fluids 25 (1982), 1922.

[8] H. E. Brandt, *Symmetry of the Complete Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, Journal of Mathematical Physics, 24 (1983), 1332, 2250.

$$S_{ij\ell}^{(s)}(k,k_1,k_2) = e_s^2 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}}{\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \left[\left(\omega_1 - \vec{k}_1\cdot\vec{v}_s\right) \frac{\partial}{\partial p_{sj}} + v_{sj}k_{1m} \frac{\partial}{\partial p_{sm}}\right] \left[\frac{\partial}{\partial p_{s\ell}} + \frac{v_{s\ell}}{\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta} k_{2n} \frac{\partial}{\partial p_{sn}}\right] f_{ps}^{R(0)} \quad . \tag{51}$$

Equations (50) and (51) are in complete accord with Tsytovich.[9] He has absorbed a factor of $\left(-e_s/\omega_1\omega_2\right)$ in equation (50) into his definition of the second-order conductivity. It is to be noted, however, that the Fourier transform and normalization conventions used by Tsytovich[9] are identical to those used here, whereas those used by Akopyan and Tsytovich[1] are not. Note also that the tensor defined by their equations[1] (18) and (20) differs from equation (51) above in that the first complex denominator $\omega - \vec{k}\cdot\vec{v}_s + i\delta$ in equation (51) here is implicitly $\omega - \vec{k}\cdot\vec{v}_s - i\delta$ there.[6,7]

Recently, some new exact symmetries of the second-order nonlinear conductivity tensor equation (51) have been discovered[3-8] and related to long-established approximate symmetries related to the Manley-Rowe relations, crossing symmetry, and the nondissipative nature of the nonlinear current. Also, a useful new polynomial representation for the tensor was obtained in which all derivatives are removed and the pole structure is clearly exhibited.[3-8] The symmetries are useful in the reduction of collective radiation probabilities.[1] Specifically the new exact symmetries are given by[6-8]

$$S_{ij\ell}^{(s)}(k_1 + k_2, k_1, k_2) + S_{i\ell j}^{(s)}(k_1 + k_2, k_2, k_1) = S_{ji\ell}^{(s)}(k_1, k_1 + k_2, k_2) - S_{j\ell i}^{(s)}(k_1, k_2, k_1 + k_2) \tag{52}$$

[1] A. V. Akopyan and V. N. Tsytovich, *Bremsstrahlung in a Nonequilibrium Plasma*, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].

[3] H. E. Brandt, *Symmetries of the Nonlinear Conductivity for a Relativistic Turbulent Plasma*, Harry Diamond Laboratories, HDL-TR-1927 (March 1981).

[4] H. E. Brandt, *Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma*, Phys. Fluids, 24 (1981), 1760.

[5] H. E. Brandt, *Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, in Plasma Astrophysics, Course and Workshop, Organized by the International School of Plasma Physics, 27 August to 7 September 1981, Varenna (Como), Italy, European Space Agency ESA SP-161 (1981) (also to be published by Pergamon Press), 361.

[6] H. E. Brandt, *On the Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, Harry Diamond Laboratories, HDL-TR-1970 (February 1982).

[7] H. E. Brandt, *Comment on Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma*, Phys. Fluids 25 (1982), 1922.

[8] H. E. Brandt, *Symmetry of the Complete Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, Journal of Mathematical Physics, 24 (1983), 1332, 2250.

[9] V. N. Tsytovich, *Nonlinear Absorption of Electromagnetic Waves During Resonant Plasma Heating*, Fiz. Plazmy 6 (1980), 1105 [Sov. J. Plasma Phys., 6 (1980), 608].

and[3-8]

$$S^{(s)}_{ij\ell}(-k_1 - k_2, k_1, k_2) + S^{(s)}_{i\ell j}(-k_1 - k_2, k_2, k_1)$$

$$= S^{(s)}_{ji\ell}(k_1, -k_1 - k_2, k_2) + S^{(s)}_{j\ell i}(k_1, k_2, -k_1 - k_2) \quad . \tag{53}$$

The approximate symmetries follow from equations (52) and (53) when resonant wave-particle interactions are negligible. For example, under this condition the well-known approximate symmetry--equation (2.83) of Tsytovich[2]--may be obtained from either equation (52) or equation (53).[3-6,8]

Proceeding to reduce the third-order polarization current using equations (42) and (28) and integrating over one of the delta functions, then

$$\vec{j}^{(s)(3)}_{dpk} = \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{e_s\vec{v}_s}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} dk_1 dk_3 dk_4 \ \delta\left(k - k_1 - k_3 - k_4\right)$$

$$\times \vec{F}_{sk_1}\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega - \omega_1 - \left(\vec{k} - \vec{k}_1\right)\cdot\vec{v}_s + i\delta\right)} \vec{F}_{sk_3}\cdot\vec{\nabla}_{p_s} \frac{1}{i\left(\omega_4 - \vec{k}_4\cdot\vec{v}_s + i\delta\right)}$$

$$\times \vec{F}_{sk_4}\cdot\vec{\nabla}_{p_s} f^{R(0)}_{p_s} \quad . \tag{54}$$

Substituting equation (34) in equation (54) and renaming wave vector variables of integration, then

---

[2] V. N. Tsytovich, *Theory of Turbulent Plasma*, Consultants Bureau, Plenum Publishing Corp. (New York, 1977).

[3] H. E. Brandt, *Symmetries of the Nonlinear Conductivity for a Relativistic Turbulent Plasma*, Harry Diamond Laboratories, HDL-TR-1927 (March 1981).

[4] H. E. Brandt, *Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma*, Phys. Fluids, 24 (1981), 1760.

[5] H. E. Brandt, *Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, in Plasma Astrophysics, Course and Workshop, Organized by the International School of Plasma Physics, 27 August to 7 September 1981, Varenna (Como), Italy, European Space Agency ESA SP-161 (1981) (also to be published by Pergamon Press), 361.

[6] H. E. Brandt, *On the Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, Harry Diamond Laboratories, HDL-TR-1970 (February 1982).

[8] H. E. Brandt, *Symmetry of the Complete Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma*, Journal of Mathematical Physics, 24 (1983), 1332. 2250.

$$\vec{j}^{(s)(3)}_{dpk} = \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{e_s^4 \vec{v}_s}{i(\omega - \vec{k}\cdot\vec{v}_s + i\delta)} dk_1 dk_2 dk_3\ \delta(k - k_1 - k_2 - k_3)$$

$$\times \left[\vec{E}_{k_1}\left(1 - \frac{\vec{v}_s\cdot\vec{k}_1}{\omega_1 + i\delta}\right) + \vec{k}_1\left(\frac{\vec{v}_s\cdot\vec{E}_{k_1}}{\omega_1 + i\delta}\right)\right]\cdot\vec{\nabla}_{p_s} \frac{1}{i[(\omega - \omega_1) - (\vec{k} - \vec{k}_1)\cdot\vec{v}_s + i\delta]}$$

$$\times \left[\vec{E}_{k_2}\left(1 - \frac{\vec{v}_s\cdot\vec{k}_2}{\omega_2 + i\delta}\right) + \vec{k}_2\left(\frac{\vec{v}_s\cdot\vec{E}_{k_2}}{\omega_2 + i\delta}\right)\right]\cdot\vec{\nabla}_{p_s} \frac{1}{i(\omega_3 - \vec{k}_3\cdot\vec{v}_s + i\delta)} \tag{55}$$

$$\times \left[\vec{E}_{k_3}\left(1 - \frac{\vec{v}_s\cdot\vec{k}_3}{\omega_3 + i\delta}\right) + \vec{k}_3\left(\frac{\vec{v}_s\cdot\vec{E}_{k_3}}{\omega_3 + i\delta}\right)\right]\cdot\vec{\nabla}_{p_s} f_{p_s}^{R(0)} \quad .$$

Equivalently then, the third-order nonlinear dynamic polarization current density--equation (55)--may be written as

$$j^{(s)(3)}_{dpki} = -e_s \int \frac{dk_1 dk_2 dk_3\ \delta(k - k_1 - k_2 - k_3)}{(\omega_1 + i\delta)(\omega_2 + i\delta)(\omega_3 + i\delta)}$$

$$\times\ \Sigma^{(s)}_{ij\ell m}(k,k_1,k_2,k_3) E_{k_1 j} E_{k_2 \ell} E_{k_3 m} \quad , \tag{56}$$

where the third-order nonlinear conductivity tensor for species s is given by

$$\Sigma^{(s)}_{ij\ell m}(k,k_1,k_2,k_3)$$

$$= -ie_s^3 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}}{\omega - \vec{k}\cdot\vec{v}_s + i\delta}$$

$$\times \left[\delta_{jn}(\omega_1 - \vec{k}_1\cdot\vec{v}_s) + k_{1n}v_{sj}\right] \frac{\partial}{\partial p_{sn}} \frac{1}{\omega - \omega_1 - (\vec{k} - \vec{k}_1)\cdot\vec{v}_s + i\delta}$$

$$\times \left[\delta_{\ell u}(\omega_2 - \vec{k}_2\cdot\vec{v}_s) + k_{2u}v_{s\ell}\right] \frac{\partial}{\partial p_{su}} \frac{1}{\omega_3 - \vec{k}_3\cdot\vec{v}_s + i\delta} \tag{57}$$

$$\times \left[\delta_{mq}(\omega_3 - \vec{k}_3\cdot\vec{v}_s) + k_{3q}v_{sm}\right] \frac{\partial}{\partial p_{sq}} f_{p_s}^{R(0)} \quad .$$

Equations (56) and (57) are also in complete accord with Tsytovich.[9] He has absorbed a factor of $(-e_s/\omega_1\omega_2\omega_3)$ into the third-order nonlinear conductivity.

---

[9] *V. N. Tsytovich, Nonlinear Absorption of Electromagnetic Waves During Resonant Plasma Heating, Fiz. Plazmy 6 (1980), 1105 [Sov. J. Plasma Phys., 6 (1980), 608].*

4. THE NONLINEAR DYNAMIC POLARIZATION FORCE

The dynamic polarization force $\vec{F}_{dp}$ on a test particle is given by the Lorentz force acting on the polarization charge accompanying the particle. Thus,

$$\vec{F}_{dp} = \sum_s \int d^3\vec{r}\left[\rho_{dp}^{(s)}(\vec{r},t)\vec{E}(\vec{r},t) + \vec{j}_{dp}^{(s)}(\vec{r},t)\times\vec{B}(\vec{r},t)\right] \quad . \tag{58}$$

The fields $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ are the electric and magnetic fields induced by the test particle interaction with the other particles in the system. It is assumed that there are no external fields. In terms of the Fourier decompositions, equation (58) becomes the following:

$$\begin{aligned}\vec{F}_{dp} &= \sum_s \int d^3\vec{r}dk_1dk_2\, e^{i(\vec{k}_1\cdot\vec{r}-\omega_1 t)}e^{i(\vec{k}_2\cdot\vec{r}-\omega_2 t)} \\ &\times \left[\rho_{dpk_1}^{(s)}\vec{E}_{k_2} + \vec{j}_{dpk_1}^{(s)}\times\vec{B}_{k_2}\right] \quad ,\end{aligned} \tag{59}$$

or performing the integral over space, then

$$\begin{aligned}\vec{F}_{dp} &= \sum_s \int dk_1dk_2\, e^{-i(\omega_1+\omega_2)t}(2\pi)^3\delta^3(\vec{k}_1 + \vec{k}_2) \\ &\times \left[\rho_{dpk_1}^{(s)}\vec{E}_{k_2} + \vec{j}_{dpk_1}^{(s)}\times\vec{B}_{k_2}\right] \quad .\end{aligned} \tag{60}$$

The time-average dynamic polarization force is given by

$$\langle\vec{F}_{dp}\rangle = \lim_{t\to\infty}\frac{1}{t}\int_{-t/2}^{t/2}\vec{F}_{dp}(t')dt' \quad . \tag{61}$$

Substituting equation (60) in equation (61) and replacing the time limits of integration by the infinite limit, then

$$\begin{aligned}\langle\vec{F}_{dp}\rangle &= \lim_{t\to\infty}\frac{1}{t}\int_{-\infty}^{\infty}dt'\sum_s\int dk_1dk_2\, e^{-i(\omega_1+\omega_2)t'}(2\pi)^3\delta^3(\vec{k}_1 + \vec{k}_2) \\ &\times \left[\rho_{dpk_1}^{(s)}\vec{E}_{k_2} + \vec{j}_{dpk_1}^{(s)}\times\vec{B}_{k_2}\right] \quad .\end{aligned} \tag{62}$$

Performing the time integration, then equation (62) becomes

$$\begin{aligned}\langle\vec{F}_{dp}\rangle &= \lim_{t\to\infty}\frac{(2\pi)^4}{t}\sum_s\int dk_1dk_2\ \delta(k_1 + k_2) \\ &\times \left[\rho_{dpk_1}^{(s)}\vec{E}_{k_2} + \vec{j}_{dpk_1}^{(s)}\times\vec{B}_{k_2}\right] \quad ,\end{aligned} \tag{63}$$

where

$$\delta(k) \equiv \delta^3(\vec{K})\delta(\omega) \quad . \tag{64}$$

Using the property of the delta function to integrate over k_1, and renaming the remaining integration variable k_2 to be k, then equation (63) becomes

$$\langle\vec{F}_{dp}\rangle = \lim_{t\to\infty} \frac{(2\pi)^4}{t} \sum_s \int dk \left[\rho^{(s)}_{dp-k}\vec{E}_k + \vec{j}^{(s)}_{dp-k}\times\vec{B}_k\right] \quad . \tag{65}$$

Using equations (12) and (31), one has

$$\rho^{(s)}_{dp-k}\vec{E}_k + \vec{j}^{(s)}_{dp-k}\times\vec{B}_k = \frac{\vec{K}\cdot\vec{j}^{(s)}_{dp-k}\vec{E}_k}{\omega - i\delta} + \frac{\vec{j}^{(s)}_{dp-k}\times\left(\vec{K}\times\vec{E}_k\right)}{\omega + i\delta} \quad . \tag{66}$$

Next using the vector identity

$$\vec{j}^{(s)}_{dp-k}\times\left(\vec{K}\times\vec{E}_k\right) = \left(\vec{j}^{(s)}_{dp-k}\cdot\vec{E}_k\right)\vec{K} - \left(\vec{j}_{dp-k}\cdot\vec{K}\right)\vec{E}_k \quad , \tag{67}$$

together with equation (12) and the fact that $\omega\delta(\omega) = 0$, then equation (66) becomes

$$\rho^{(s)}_{dp-k}\vec{E}_k + \vec{j}^{(s)}_{dp-k}\times\vec{B}_k = \frac{\vec{K}\left(\vec{E}_k\cdot\vec{j}^{(s)}_{dp-k}\right)}{\omega + i\delta} \quad . \tag{68}$$

Therefore, substituting equation (68) in equation (65) one has

$$\langle\vec{F}_{dp}\rangle = \lim_{t\to\infty} \frac{(2\pi)^4}{t} \sum_s \int dk\ \vec{K}\ \frac{\vec{E}_k\cdot\vec{j}^{(s)}_{dp-k}}{\omega + i\delta} \quad . \tag{69}$$

Substituting equation (41) in equation (69), then

$$\langle\vec{F}_{dp}\rangle = \sum_{n=0}^{\infty} \vec{F}^{(n)}_{dp} \quad , \tag{70}$$

where

$$\vec{F}^{(n)}_{dp} = \lim_{t\to\infty} \frac{(2\pi)^4}{t} \sum_s \int dk\ \frac{\vec{K}}{\omega + i\delta}\ \vec{E}_k\cdot\vec{j}^{(s)(n+1)}_{dp-k} \quad . \tag{71}$$

Next, substituting equation (46) in equation (71), then

$$\vec{F}^{(0)}_{dp} = \lim_{t\to\infty} \frac{(2\pi)^4}{t} \sum_s \int \frac{dk}{\omega + i\delta} \vec{k}\, E_{k_i} \sigma^{(s)}_{i\ell}(-k) E_{-k\ell} \quad . \tag{72}$$

Replacing the variable of integration k by -k in equation (72), and realizing that because the fields are real

$$\vec{E}_{-k} = \vec{E}^{*}_{k} \quad , \tag{73}$$

then equation (72) becomes

$$\vec{F}^{(0)}_{dp} = \lim_{t\to\infty} \frac{(2\pi)^4}{t} \sum_s \int \frac{dk}{\omega - i\delta} \vec{k}\, E^{*}_{k_i} E_{k_\ell} \sigma^{(s)}_{i\ell}(k) \quad . \tag{74}$$

An alternative form is obtained by introducing an integrated delta function in equation (72) to obtain an equivalent expression, namely,

$$\vec{F}^{(0)}_{dp} = \lim_{t\to\infty} \frac{(2\pi)^4}{t} \sum_s \int \frac{dk\,dk_1}{\omega + i\delta}\, \delta\left(k + k_1\right)\vec{k}\, E_{ki} E_{k_1\ell} \sigma^{(s)}_{i\ell}\left(k_1\right) \quad . \tag{75}$$

Next substituting equation (50) in equation (71), and noting that the delta function is even, one obtains

$$\vec{F}^{(1)}_{dp} = -\lim_{t\to\infty} \frac{(2\pi)^4}{t} \sum_s e_s \int \frac{dk\,dk_1 dk_2 \;\; \delta\left(k + k_1 + k_2\right)}{(\omega + i\delta)\left(\omega_1 + i\delta\right)\left(\omega_2 + i\delta\right)} \times E_{ki} E_{k_1 j} E_{k_2 \ell} \vec{k}\, S^{(s)}_{ij\ell}\left(-k,k_1,k_2\right) \quad . \tag{76}$$

Equation (76) is in complete agreement with equation (18) of Akopyan and Tsytovich[1] since from equation (51) as noted earlier it follows that

$$S^{(s)}_{ij\ell}\left(-k,k_1,k_2\right) = -\bar{S}^{(s)}_{ij\ell}\left(k,k_1,k_2\right) \quad , \tag{77}$$

where $\bar{S}^{(s)}_{ij\ell}\left(k,k_1,k_2\right)$ designates the conductivity tensor appearing in Akopyan and Tsytovich[1] in which evidently the first complex denominator is implicitly $\omega - \vec{k}\cdot\vec{v}_s - i\delta$. This may be seen from equation (18) there where, because of the delta function, one has effectively $\bar{S}^{(s)}_{ij\ell}\left(-k_1 - k_2,k_1,k_2\right)$. Assuming that

---

[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy,* _1_ *(1975), 673 [Sov. J. Plasma Phys.,* _1_ *(1975), 371].*

equation (50) holds there also, except for the differing Fourier transform convention, then because of the delta function only $S^{(s)}_{ij\ell}(k_1 + k_2, k_1, k_2)$ can enter. Equation (18) of Akopyan and Tsytovich[1] has an additional factor of $(2\pi)^{-9}(2\pi)^{-3}$ which apparently arises from different Fourier transform and background normalization conventions. For example, in their equation (5)[1] the Fourier transform convention employed has a factor of $(2\pi)^{-3}$ in the inverse Fourier transform in the integration over the three-dimensional wave vector space, and a factor of 1 for the integration over frequency, giving a total factor of $(2\pi)^{-3}$, whereas here a total factor of 1 is used as in equation (9), for example. Also, the Fourier transform itself has a factor of $(2\pi)^{-1}$ there and $(2\pi)^{-4}$ here as in equation (21), for example. There is also another additional factor of $(2\pi)^{-3}$ in their equation (18)[1] which is apparently due to differing background normalization. Because of the different Fourier transform convention alone, the counterpart of their equation (18)[1] would have an additional factor of $(2\pi)^3$. Apparently it has been absorbed into the normalization of $f^{R(0)}_{ps}$ there. In short, the $f^{R(0)}_{ps}$ there must be $(2\pi)^3$ times that here. Alternatively if the normalization is in fact the same as that here, then there is an erroneous factor of $(2\pi)^{-3}$ appearing there. Also the factor of 1/6 in Akopyan and Tsytovich[1] arises from the explicit symmetrization chosen there.

Next substituting equation (56) in equation (71) and noting that the delta function is even, one obtains

$$\vec{F}^{(2)}_{dp} = -\lim_{t\to\infty} \frac{(2\pi)^4}{t} \sum_s e_s \int \frac{dk\,dk_1 dk_2 dk_3\ \delta(k + k_1 + k_2 + k_3)}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_2 + i\delta)(\omega_3 + i\delta)}$$
$$\times E_{ki}E_{k_1 j}E_{k_2 \ell}E_{k_3 m}\ \vec{k}\ \Sigma^{(s)}_{ij\ell m}(-k, k_1, k_2, k_3) \quad . \tag{78}$$

It is to be noted that equations (78) and (57) are in apparent agreement with equations (19) and (21) of Akopyan and Tsytovich.[1] Evidently in the conductivity tensor $T^{(s)}_{ij\ell m}(k, k_1, k_2, k_3)$ given by equation (21) there,[1] the first two complex denominators are implicitly $\omega - \vec{k}\cdot\vec{v}_s - i\delta$ and $\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_s - i\delta$, respectively, and also there is no overall factor of i as there is in equation (57) here. Therefore

$$\Sigma^{(s)}_{ij\ell m}(-k, k_1, k_2, k_3) = -iT^{(s)}_{ij\ell m}(k, k_1, k_2, k_3) \quad . \tag{79}$$

[1]A. V. Akopyan and V. N. Tsytovich, *Bremsstrahlung in a Nonequilibrium Plasma*, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].

Thus, comparing equations (78) and (79) with equation (19) of Akopyan and Tsytovich,[1] one finds that they are in complete agreement. Again the additional factors of $(2\pi)^{-12}$ and $(2\pi)^{-3}$ and 1/24 in their equation (19)[1] apparently arise from differing Fourier transform and background normalization conventions and explicit symmetrization, respectively.

In summary, then, using equation (70), the time-average dynamic polarization force on a relativistic test particle to fourth order, in the field in a nonequilibrium beam-plasma system for a slowly varying, nearly spatially independent background with no external fields, is given by

$$\langle \vec{F}_{dp} \rangle = \vec{F}^{(0)}_{dp} + \vec{F}^{(1)}_{dp} + \vec{F}^{(2)}_{dp} \quad . \tag{80}$$

The linear, second-order nonlinear, and third-order nonlinear dynamic polarization forces $\vec{F}^{(0)}_{dp}$, $\vec{F}^{(1)}_{dp}$, and $\vec{F}^{(2)}_{dp}$ are given by equations (75), (76), and (78), respectively.

5. CONCLUSION

An expression--equations (80), (75), (76), and (78)--has been obtained for the time-averaged dynamic polarization force on a relativistic test particle to fourth order in the total field in a nonequilibrium beam-plasma system for a slowly varying, nearly spatially independent background with no external fields. This result has been used in the work of Akopyan and Tsytovich in the theory of collective bremsstrahlung in nonequilibrium plasmas.

The present work together with related work by the author[10,*,†,‡] is important for ongoing work in calculating collective radiation processes and conditions for the occurrence of radiative instability in relativistic beam-plasma systems.

---

[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

[10] *H. E. Brandt, The Gluckstern-Hull Formula for Electron-Nucleus Bremsstrahlung, Harry Diamond Laboratories, HDL-TR-1884 (May 1980)*

[*] *H. E. Brandt, Theoretical Methods in the Calculation of the Bremsstrahlung Recoil Force in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-83-6 (April 1983), to be published as HDL-TR-2009.*

[†] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam Plasma System, Harry Diamond Laboratories, HDL-PRL-82-8 (June 1982), to be published as HDL-TR-1996.*

[‡] *Other related work, prepared in preprint form, will be published later and is available from the author.*

LITERATURE CITED

(1) A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].

(2) V. N. Tsytovich, Theory of Turbulent Plasma, Consultants Bureau, Plenum Publishing Corp. (New York, 1977).

(3) H. E. Brandt, Symmetries of the Nonlinear Conductivity for a Relativistic Turbulent Plasma, Harry Diamond Laboratories, HDL-TR-1927 (March 1981).

(4) H. E. Brandt, Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma, Phys. Fluids, 24 (1981), 1760.

(5) H. E. Brandt, Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma, in Plasma Astrophysics, Course and Workshop, Organized by the International School of Plasma Physics, 27 August to 7 September 1981, Varenna (Como), Italy, European Space Agency ESA SP-161 (1981) (also to be published by Pergamon Press), 361.

(6) H. E. Brandt, On the Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma, Harry Diamond Laboratories, HDL-TR-1970 (February 1982).

(7) H. E. Brandt, Comment on Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma, Phys. Fluids 25 (1982), 1922.

(8) H. E. Brandt, Symmetry of the Complete Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma, J. Math. Phys. 24 (1983), 1332, 2250.

(9) V. N. Tsytovich, Nonlinear Absorption of Electromagnetic Waves During Resonant Plasma Heating, Fiz. Plazmy 6 (1980), 1105 [Sov. J. Plasma Phys., 6 (1980), 608].

(10) H. E. Brandt, The Gluckstern-Hull Formula for Electron-Nucleus Bremsstrahlung, Harry Diamond Laboratories, HDL-TR-1884 (May 1980).

DISTRIBUTION

ADMINISTRATOR
DEFENSE TECHNICAL INFORMATION CENTER
ATTN DTIC-DDA (12 COPIES)
CAMERON STATION, BUILDING 5
ALEXANDRIA, VA 22314

COMMANDER
US ARMY MATERIEL DEVELOPMENT & READINESS
COMMAND
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

DIRECTOR
DEFENSE ADVANCED RESEARCH PROJECTS
AGENCY
ATTN R. GULLICKSON
ATTN L. C. MARQUET
ATTN R. SEPUCHA
1400 WILSON BLVD
ARLINGTON, VA 22209

DIRECTOR
DEFENSE COMMUNICATIONS AGENCY
WASHINGTON, DC 20305

DIRECTOR
DEFENSE INTELLIGENCE AGENCY
ATTN D. SPOHN
ATTN DT-1, NUCLEAR & APPLIED
SCIENCES DIV
ATTN ELECTRONIC WARFARE BRANCH
WASHINGTON, DC 20301

UNDER SECRETARY OF DEFENSE
FOR RESEARCH & ENGINEERING
ATTN DEPUTY UNDER SECRETARY
(RESEARCH & ADVANCED TECH)
ATTN D. L. LAMBERSON
WASHINGTON, DC 20301

OFFICE OF THE UNDER SECRETARY OF
DEFENSE FOR RESEARCH & ENGINEERING
DIR ENERGY TECHNOLOGY OFFICE
THE PENTAGON
WASHINGTON, DC 20301

DIRECTOR
DEFENSE NUCLEAR AGENCY
ATTN DEPUTY DIRECTOR
SCIENTIFIC TECHNOLOGY
ATTN G. BAKER
ATTN RAEV, ELECTRONIC VULNERABILITY
ATTN J. Z. FARBER
ATTN G. K. SOPER
ATTN V. VAN LINT
WASHINGTON, DC 20305

DIRECTOR
NATIONAL SECURITY AGENCY
ATTN TECHNICAL LIBRARY
ATTN F. BEDARD
FT MEADE, MD 20755

ASSISTANT SECRETARY OF THE ARMY (RDA)
ATTN DEP FOR SCI & TECH
WASHINGTON, DC 20301

OFFICE, DEPUTY CHIEF OF STAFF
FOR OPERATIONS & PLANS
DEPT OF THE ARMY
ATTN DAMO-SSN, NUCLEAR DIV
WASHINGTON, DC 20310

OFFICE OF THE DEPUTY CHIEF OF STAFF
FOR RESEARCH, DEVELOPMENT,
& ACQUISITION
DEPARTMENT OF THE ARMY
ATTN DIRECTOR OF ARMY RESEARCH,
M. E. LASSER
ATTN DAMA-CSS-N, NUCLEAR TEAM
ATTN DAMA-ARZ-O, F. D. VERDERAME
WASHINGTON, DC 20310

COMMANDER
US ARMY ABERDEEN PROVING GROUND
ATTN STEAP-TL, TECH LIB
ABERDEEN PROVING GROUND, MD 21005

COMMANDER
BALLISTIC MISSILE DEFENSE
ADVANCED TECHNOLOGY CENTER
ATTN D. SCHENK
PO BOX 1500
HUNTSVILLE, AL 35807

DIRECTOR
US ARMY BALLISTIC RESEARCH LABORATORY
ATTN DRDAR-TSB-S (STINFO)
ATTN D. ECCLESHALL
ATTN C. HOLLANDSWORTH
ATTN R. KREMENS
ABERDEEN PROVING GROUND, MD 21005

US ARMY ELECTRONICS TECHNOLOGY &
DEVICES LABORATORY
ATTN DELET-DD
ATTN N. WILSON
FT MONMOUTH, NJ 07703

US ARMY ERADCOM
ATTN C. M. DESANTIS
FT MONMOUTH, NJ 07703

DISTRIBUTION (Cont'd)

COMMANDER
US ARMY FOREIGN SCIENCE
& TECHNOLOGY CENTER
FEDERAL OFFICE BLDG
ATTN DRXST-SD, SCIENCES DIV
ATTN T. CALDWELL
220 7TH STREET, NE
CHARLOTTESVILLE, VA 22901

DIRECTOR
US ARMY MATERIEL SYSTEMS
ANALYSIS ACTIVITY
ATTN DRXSY-MP
ABERDEEN PROVING GROUND, MD 21005

COMMANDER
US ARMY MATERIALS & MECHANICS
RESEARCH CENTER
ATTN DRXMR-H, BALLISTIC MISSILE
DEF MATLS PROG OFC
WATERTOWN, MA 02172

COMMANDER
US ARMY MISSILE COMMAND
ATTN DRDMI-TR, PHYSICAL SCIENCES DIR
REDSTONE ARSENAL, AL 35809

COMMANDER
US ARMY MISSILE & MUNITIONS
CENTER & SCHOOL
ATTN ATSK-CTD-F
REDSTONE ARSENAL, AL 35809

ARMY RESEARCH OFFICE (DURHAM)
PO BOX 12211
ATTN H. ROBL
ATTN R. LONTZ
ATTN B. D. GUENTHER
ATTN TECH LIBRARY
RESEARCH TRIANGLE PARK, NC 27709

COMMANDER
US ARMY RSCH & STD GP (EUR)
ATTN CHIEF, PHYSICS & MATH BRANCH
FPO NEW YORK 09510

NAVAL AIR SYSTEMS COMMAND
ATTN LCDR G. BATES, PMS-405
WASHINGTON, DC 20361

NAVAL INTELLIGENCE SUPPORT CENTER
ATTN M. KOONTZ
4301 SUITLAND RD
SUITLAND, MD 20390

NAVAL MATERIAL COMMAND
ATTN T. HORWATH
2211 JEFFERSON DAVIS HWY
WASHINGTON, DC 20301

OFFICE OF NAVAL RESEARCH
ATTN C. ROBERSON
ATTN W. J. CONDELL
800 N. QUINCY ST
ARLINGTON, VA 22217

SUPERINTENDENT
NAVAL POSTGRADUATE SCHOOL
ATTN LIBRARY, CODE 2124
MONTEREY, CA 93940

DIRECTOR
NAVAL RESEARCH LABORATORY
ATTN 2600, TECHNICAL INFO DIV
ATTN 5540, LASER PHYSICS
ATTN 6000, MATL & RADIATION SCI & TE
ATTN B. RIPIN
ATTN L. A. COSBY
ATTN E. E. KEMPE
ATTN J. T. SCHRIEMPF
ATTN R. F. WENZEL
ATTN R. HETTCHE
ATTN J. GOLDEN
ATTN V. L. GRANATSTEIN
ATTN M. A. REED
ATTN R. K. PARKER
ATTN P. A. SPRANGLE
ATTN S. GOLD
ATTN C. A. KAPETANAKOS
ATTN S. AHN
ATTN T. P. COFFEY
ATTN R. JACKSON
ATTN I. M. VITKOVITSKY
ATTN M. FRIEDMAN
ATTN J. PASOUR
ATTN G. COOPERSTEIN
WASHINGTON, DC 20375

COMMANDER
NAVAL SURFACE WEAPONS CENTER
ATTN V. PUGLIELLI
ATTN DX-21, LIBRARY DIV
DAHLGREN, VA 22448

COMMANDER
NAVAL SURFACE WEAPONS CENTER
ATTN J. Y. CHOE
ATTN F. SAZAMA
ATTN H. UHM
ATTN V. KENYON
ATTN E. NOLTING
ATTN WA-13, HIGH-ENERGY LASER BR
ATTN WA-50, NUCLEAR WEAPONS
EFFECTS DIV
ATTN WR, RESEARCH & TECHNOLOGY DEPT
ATTN WR-40, RADIATION DIV
ATTN E-43, TECHNICAL LIB
WHITE OAK, MD 20910

COMMANDER
NAVAL WEAPONS CENTER
ATTN 315, LASER/INFRARED SYS DIV
ATTN 381, PHYSICS DIV
CHINA LAKE, CA 93555

HQ USAF/SAMI
WASHINGTON, DC 20330

ASSISTANT SECRETARY OF THE AIR FORCE
(RESEARCH & DEVELOPMENT)
WASHINGTON, DC 20330

DIRECTOR
AF OFFICE OF SCIENTIFIC RESEARCH
BOLLING AFB
ATTN NP, DIR OF PHYSICS
ATTN M. A. STROSCIO
ATTN R. BARKER
WASHINGTON, DC 20332

COMMANDER
AF WEAPONS LAB, AFSC
ATTN J. GENEROSA
ATTN A. H. GUENTHER
ATTN W. E. PAGE
ATTN LR, LASER DEV DIV
KIRTLAND AFB, NM 87117

WRIGHT-PATTERSON AFB
FOREIGN TECHNOLOGY DIVISION/ETD
ATTN J. BUTLER
WRIGHT-PATTERSON AFB, OH 45433

CENTRAL INTELLIGENCE AGENCY
ATTN R. PETTIS
ATTN D. B. NEWMAN
PO BOX 1925
WASHINGTON, DC 20013

DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
ATTN LIBRARY
ATTN CENTER FOR RADIATION RESEARCH
ATTN C. TEAGUE
ATTN E. MARX
WASHINGTON, DC 20234

DEPARTMENT OF ENERGY
ATTN T. F. GODLOVE
ATTN M. MURPHY
ATTN A. COLE
ATTN O. P. MANLEY
WASHINGTON, DC 20585

NATIONAL AERONAUTICS & SPACE
ADMINISTRATION
ATTN R. RAMATY
GOODARD SPACE FLIGHT CENTER
GREENBELT, MD 20771

ADMINISTRATOR
NASA HEADQUARTERS
WASHINGTON, DC 20546

DIRECTOR
NASA
GODDARD SPACE FLIGHT CENTER
ATTN 250, TECH INFO DIV
GREENBELT, MD 20771

AMES LABORATORY (ERDA)
IOWA STATE UNIVERSITY
ATTN NUCLEAR SCIENCE CATEGORY
AMES, IA 50011

ASTRONOMICAL INSTITUTE--ZONNENBURG 2
ATTN M. KUPERUS
3512 NL UTRECHT, THE NETHERLANDS

THE AUSTRALIAN NATIONAL UNIVERSITY
DEPT OF THEORETICAL PHYSICS
ATTN D. B. MELROSE
PO BOX 4
CANBERRA A.C.T. 2600
AUSTRALIA

B. K. DYNAMICS
ATTN R. J. LINZ
15825 SHADY GROVE RD
ROCKVILLE, MD 20850

BOEING AEROSPACE
ATTN H. T. BUSHER
PO BOX 3999 MS 8R-10
SEATTLE, WA 98124

BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC
ATTN PHYSICS DEPT
UPTON, LONG ISLAND, NY 11973

CALIFORNIA INSTITUTE OF TECHNOLOGY
THEORETICAL ASTROPHYSICS
ATTN K. THORNE
130-33
PASADENA, CA 91125

UNIVERSITY OF CALIFORNIA
DEPT OF APPL PHYS & INFORMATION
SCIENCE
ATTN H. ALFVEN
LA JOLLA, CA 92093

CALIFORNIA INSTITUTE OF TECHNOLOGY
JET PROPULSION LABORATORY
ATTN J. C. HUBBARD
IRVINE, CA 92717

DISTRIBUTION (Cont'd)

UNIVERSITY OF CALIFORNIA
ELECTRICAL ENGINEERING DEPARTMENT
ATTN C. K. BIRDSALL
BERKELEY, CA 94720

UNIVERSITY OF CALIFORNIA
PHYSICS DEPT
ATTN A. N. KAUFMAN
BERKELEY, CA 94720

UNIVERSITY OF CALIFORNIA, DAVIS
APPLIED SCIENCES
ATTN J. DEGROOT
DAVIS, CA 95616

UNIVERSITY OF CALIFORNIA, IRVINE
DEPT OF PHYSICS
ATTN G. BENFORD
ATTN N. ROSTOKER
ATTN M. MAYER
ATTN A. RAY
IRVINE, CA 92717

UNIVERSITY OF CALIFORNIA, LOS ANGELES
DEPT OF PHYSICS
ATTN K. NOZAKI
LOS ANGELES, CA 90025

CAMBRIDGE UNIVERSITY
INSTITUTE OF ASTRONOMY
MADINGLEY ROAD
ATTN M. REES
CAMBRIDGE CB 3 0 HA, ENGLAND

CHALMERS UNIV OF TECHNOLOGY
INST OF ELECTROMAGNETIC FIELD
THEORY
ATTN H. WILHELMSSON
S-41296 GOTHENBURG, SWEDEN

UNIVERSITY OF CHICAGO
LAB FOR ASTROPHYSICS & SPACE RESEARCH
ATTN E. PARKER
CHICAGO, IL 60637

UNIVERSITY OF COLORADO
DEPT OF ASTROGEOPHYSICS
ATTN M. GOLDMAN
ATTN D. SMITH
BOULDER, CO 80309

COLUMBIA UNIVERSITY
ATTN S. JOHNSTON
216 MUDD BLDG
NEW YORK, NY 10027

CORNELL UNIVERSITY
ATTN R. LOVELACE
ATTN R. N. SUDAN
ATTN J. NATION
ATTN D. HAMMER
ATTN H. FLEISHMANN
ITHACA, NY 14853

DARTMOUTH COLLEGE
PHYSICS DEPT
ATTN J. E. WALSH
HANOVER, NH 03755

UNIVERSITY OF THE DISTRICT
OF COLUMBIA
VAN NESS CAMPUS
DEPT OF PHYSICS
ATTN M. J. SMITH
4200 CONNECTICUT AVE, NW
WASHINGTON, DC 20008

ENGINEERING SOCIETIES LIBRARY
345 EAST 47TH STREET
ATTN ACQUISITIONS DEPARTMENT
NEW YORK, NY 10017

GENERAL DYNAMICS
ATTN K. H. BROWN
PO BOX 2507 MZ 44-21
POMONA, CA 91769

UNIVERSITY OF ILLINOIS AT
URBANA--CHAMPAIGN
DEPT OF PHYSICS
ATTN N. IWAMOTO
URBANA, IL 61801

ISTITUTO DI FISICA DELL' UNIVERSITA
VIA CELORIA 16
ATTN P. CALDIROLA
ATTN C. PAIZIS
ATTN E. SINDONI
20133 MILANO, ITALY

ISTITUTO DI FISICA DELL' UNIVERSITA
ATTN A. CAVALIERE
ATTN R. RUFFINI
ROME, ITALY

ISTITUTO DI FISICA GENERALE
DELL' UNIVERSITA
CORSO M. D'AZEGLIO
ATTN A. FERRARI
46 TORINO, ITALY

JAYCOR
ATTN E. CONRAD
205 S. WHITING ST
ALEXANDRIA, VA 22304

KAMAN SCIENCES CORP
ATTN T. A. TUMOLILLO
1500 GARDEN OF THE GODS
COLORADO SPRINGS, CO 80907

LAWRENCE BERKELEY LAB
ATTN A. FALTENS
ATTN A. M. SESSLER
ATTN D. KEEFE
BERKELEY, CA 94720

LAWRENCE LIVERMORE LABORATORY
UNIVERSITY OF CALIFORNIA
ATTN L. MARTIN
ATTN P. WHEELER
ATTN H. S. CABAYAN
ATTN R. BRIGGS
ATTN E. K. MILLER
ATTN R. ZIOLKOWSKI
ATTN R. SCARPETTI
ATTN R. MINICH
ATTN S. BURKHART
ATTN G. VOGTLIN
ATTN V. W. SLIVINSKY
ATTN S. L. YU
ATTN G. CRAIG
ATTN J. WYATT
ATTN R. ALVAREZ
ATTN G. LASCHE
ATTN J. H. YEE
ATTN H. W. MELDNER
ATTN J. S. PETTIBONE
ATTN A. J. POGGIO
ATTN M. LOYD
PO BOX 808
LIVERMORE, CA 94550

CHIEF
LIVERMORE DIVISION, FIELD COMMAND, DNA
LAWRENCE LIVERMORE LABORATORY
PO BOX 808
LIVERMORE, CA 94550

LOS ALAMOS SCI LAB
ATTN S. COLGATE
ATTN D. F. DUBOIS
ATTN D. FORSLAND
ATTN C. M. FOWLER
ATTN B. WARNER
ATTN J. LANDT
ATTN R. HOEBERLING
ATTN T. R. KING
ATTN A. KADISH
ATTN K. LEE
ATTN R. W. FREYMAN
ATTN A. W. CHURMATZ
ATTN L. M. DUNCAN
ATTN D. B. HENDERSON
ATTN L. E. THODE
ATTN H. A. DAVIS
ATTN T. KWAN
PO BOX 1663
LOS ALAMOS, NM 87545

UNIVERSITY OF MARYLAND
DEPT OF ELECTRICAL ENGINEERING
ATTN M. REISER
ATTN W. DESTLER
ATTN C. D. STRIFFLER
ATTN M. T. RHEE
COLLEGE PARK, MD 20742

UNIVERSITY OF MARYLAND
DEPT OF PHYSICS & ASTRONOMY
ATTN H. R. GRIEM
ATTN E. OTT
ATTN C. GREBOGI
ATTN K. PAPADOPOULOS
ATTN J. WEBER
COLLEGE PARK, MD 20742

MASSACHUSETTS INSTITUTE
OF TECHNOLOGY
PLASMA FUSION CENTER
ATTN G. BEKEFI
ATTN J. BELCHER
ATTN T. CHANG
ATTN B. COPPI
ATTN R. DAVIDSON
ATTN C. LIN
ATTN S. OLBERT
ATTN B. ROSSI
CAMBRIDGE, MA 02139

MAX PLANCK INST, FUR AERONOMIE
3411 KATLENBG
ATTN J. AXFORD
ATTN E. MARSCH
LINDAU 3, WEST GERMANY

MAX PLANCK INSTITUTE OF PHYSICS
& ASTROPHYSICS
ATTN M. NORMAN
ATTN J. TRUMPER
GARCHING BEI MUNICH, WEST GERMANY

MAXWELL LABORATORIES
ATTN J. S. PEARLMAN
8835 BALBOA AVE
SAN DIEGO, CA 92123

MISSION RESEARCH CORP
ATTN B. GODFREY
ATTN D. J. SULLIVAN
ATTN M. BOLLEN
ATTN C. LONGMIRE
ATTN D. VOSS
1720 RANDOLPH RD, SE
ALBUQUERQUE, NM 87106

UNIVERSITY OF NAGOYA
DEPT OF PHYSICS
ATTN S. HAYAKAWA
NAGOYA, JAPAN

DISTRIBUTION (Cont'd)

NATIONAL RESEARCH COUNCIL
DIVISION OF PHYSICS
ATTN P. JAANIMAGI
OTTOWA, ONTARIO
CANADA

NATIONAL SCIENCE FOUNDATION
DIVISION OF ASTRONOMICAL SCIENCES
ATTN M. AIZENMAN
1800 G STREET, NW
WASHINGTON, DC 20550

OAK RIDGE NATIONAL LABORATORY
PO BOX Y
ATTN A. C. ENGLAND
OAK RIDGE, TN 37830

OCCIDENTAL RESEARCH CORP
ATTN D. B. CHANG
2100 SE MAIN
IRVINE, CA 92713

OXFORD UNIVERSITY
DEPARTMENT THEORETICAL PHYSICS
1 KEBLE RD
ATTN D. TER HAAR
OXFORD OX1 3NP, ENGLAND

OXFORD UNIVERSITY
ASTROPHYSICS DEPARTMENT
SOUTH PARK ROAD
ATTN A. HALL
OXFORD--OXI 3 RQ, ENGLAND

OXFORD UNIVERSITY
DEPT OF ENGINEERING SCIENCE
PARKS ROAD
ATTN L. M. WICKENS
ATTN J. E. ALLEN
OXFORD, UNITED KINGDOM

PHYSICS INTERNATIONAL
ATTN B. A. LIPPMANN
ATTN R. D. GENUARIO
2700 MERCER ST
SAN LEANDRO, CA 94577

PRINCETON UNIVERSITY
ASTROPHYSICAL SCIENCES
PETON HALL
ATTN R. KULSRUD
PRINCETON, NJ 08540

PRINCETON UNIVERSITY
PLASMAPHYSICS LABORATORY
ATTN R. WHITE
PRINCETON, NJ 08540

QUEEN MARY COLLEGE
DEPT OF APPLIED MATH
MILE END ROAD
ATTN D. BURGESS
LONDON EI 4NS, ENGLAND

RUHR UNIVERSITY
ATTN W. SEBOLD
ATTN W. ZWINGMANN
4630 BOCHUM 1, WEST GERMANY

INSTITUTE OF EXPERIMENTAL PHYSICS
RUHR-UNIVERSITY
POSTFACH 2148
ATTN H. KUNZE
436 BOCHUM, WEST GERMANY

SACHS/FREEMAN ASSOC
ATTN R. A. MAHAFFEY
19300 GALLANT FOX
SUITE 214
BOWIE, MD 20715

SANDIA LABORATORIES
ATTN R. B. MILLER
ATTN J. POWELL
ATTN W. BALLARD
ATTN C. EKDAHL
ATTN W. D. BROWN
ATTN S. HUMPHRIES, JR.
ATTN G. ROHWEIN
ATTN K. PRESTWICH
4244 SANDIA LABS
ALBUQUERQUE, NM 87115

SCIENCE APPLICATIONS, INC
ATTN E. P. CORNET
ATTN A. DROBOT
ATTN E. KANE
1710 GOODRIDGE DR
PO BOX 1303
MCLEAN, VA 22012

SCIENCE APPLICATIONS, INC
ATTN F. CHILTON
1200 PROSPECT ST
PO BOX 2351
LA JOLLA, CA 92038

SERVICE DE CHEMIE PHYSIQUELL
ATTN R. BALESCU
CAMPUS PLAINE U.L.B.
CODE POSTAL n° 231
BOULEVARD DU TRIOMPHE
10 50 BRUXELLES, BELGIUM

SPECOLA VATICANA
ATTN W. STOEGER, S. J.
I-00120 CITTA DEL VATICANO
ITALY

DISTRIBUTION (Cont'd)

SRI INTERNATIONAL
ATTN G. AUGUST
ATTN C. L. RINO
333 RAVENSWOOD AVE
MENLO PARK, CA 94025

STANFORD UNIVERSITY
INST PLASMA RES
ATTN P. A. STURROCK
STANFORD, CA 94305

STERRERWACHT-LEIDEN
ATTN C. A. NORMAN
ATTN P. ALLAN
2300 RA LEIDEN, THE NETHERLANDS

TEL-AVIV UNIVERSITY
DEPT OF PHYSICS & ASTRONOMY
ATTN G. TAUBER
TEL AVIV, ISRAEL

TELEDYNE BROWN ENGINEERING
CUMMINGS RESEARCH PARK
ATTN MELVIN L. PRICE, MS-44
HUNTSVILLE, AL 35807

INSTITUTE FOR THEORETICAL MECHANICS-
RUG
KRIJGSLAAN 271-S9
ATTN F. VERHEEST
B-9000 GENT, BELGIUM

UNIVERSITY OF TENNESSEE
DEPT OF ELECTRICAL ENGINEERING
ATTN I. ALEXEFF
ATTN J. R. ROTH
KNOXVILLE, TN 37916

UMEA UNIVERSITY
DEPT OF PLASMA PHYSICS
ATTN J. LARSSON
ATTN L. STENFLO
S-90187 UMEA
SWEDEN

UNIVERSITY OF WASHINGTON
DEPT OF PHYSICS
ATTN M. BAKER
SEATTLE, WA 98195

WEIZMANN INSTITULE
DEPT OF NUCLEAR PHYSICS
ATTN AMRI WANDEL
REHOVOT, ISRAEL

WESTERN RESEARCH CORP
ATTN R. C. HUNTER
225 BROADWAY, SUITE 1600
SAN DIEGO, CA 92101

US ARMY ELECTRONICS RESEARCH &
DEVELOPMENT COMMAND
ATTN COMMANDER, DRDEL-CG
ATTN TECHNICAL DIRECTOR, DRDEL-CT
ATTN PUBLIC AFFAIRS OFFICE, DRDEL-IN

HARRY DIAMOND LABORATORIES
ATTN CO/TD/TSO/DIVISION DIRECTORS
ATTN RECORD COPY, 81200
ATTN HDL LIBRARY, 81100 (3 COPIES)
ATTN HDL LIBRARY (WOODBRIDGE), 81100
ATTN TECHNICAL REPORTS BRANCH, 81300
ATTN LEGAL OFFICE, 97000
ATTN CHAIRMAN, EDITORIAL COMMITTEE
ATTN MORRISON, R. E., 13500
ATTN CHIEF, 21000
ATTN CHIEF, 21100
ATTN CHIEF, 21200
ATTN CHIEF, 21300
ATTN CHIEF, 21400
ATTN CHIEF, 21500
ATTN CHIEF, 22000
ATTN CHIEF, 22100
ATTN CHIEF, 22300
ATTN CHIEF, 22800
ATTN CHIEF, 22900
ATTN CHIEF, 20240
ATTN CHIEF, 11000
ATTN CHIEF, 13000
ATTN CHIEF, 13200
ATTN CHIEF, 13300
ATTN CHIEF, 13500
ATTN CHIEF, 15200
ATTN BROWN, E., 00211
ATTN SINDORIS, A., 00211
ATTN GERLACH, H., 11100
ATTN LIBELO, L., 11200
ATTN LOKERSON, D., 11400
ATTN CROWNE, F., 13200
ATTN DROPKIN, H., 13200
ATTN LEAVITT, R., 13200
ATTN MORRISON, C., 13200
ATTN SATTLER, J., 13200
ATTN WORTMAN, D., 13200
ATTN KULPA, S., 13300
ATTN SILVERSTEIN, J., 13300
ATTN FAZI, C., 13500
ATTN LOMONACO, S., 15200
ATTN J. CORRIGAN, 20240
ATTN FARRAR, F., 21100
ATTN GARVER, R., 21100
ATTN TATUM, J., 21100
ATTN MERKEL, G., 21300
ATTN MCLEAN, B., 22300
ATTN OLDHAM, T., 22300
ATTN BLACKBURN, J., 22800
ATTN GILBERT, R., 22800
ATTN KLEBERS, J., 22800
ATTN VANDERWALL, J., 22800
ATTN BROMBORSKY, A., 22900

DISTRIBUTION (Cont'd)

HARRY DIAMOND LABORATORIES (Cont'd)
ATTN DAVIS, D., 22900
ATTN GRAYBILL, S., 22900
ATTN HUTTLIN, G. A., 22900
ATTN KEHS, A., 22900
ATTN KERRIS, K., 22900
ATTN LAMB, R., 22900
ATTN LINDSAY, D., 22900
ATTN LITZ, M., 22900
ATTN RUTH, B., 22900
ATTN STEWART, A., 22900
ATTN SOLN, J., 22900
ATTN WHITTAKER, D., 22900
ATTN ELBAUM, S., 97100
ATTN BRANDT, H. E., 22300
(60 copies)

END

FILMED

2-84

DTIC